

Resource-adaptive distributed stabilizer measurement with very noisy Bell pairs

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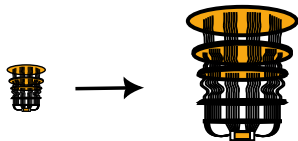
I'll be hiring

Work together with:

Moritz Schmidt, Martin Moureau, Benjamin Rodatz, Boldizsár Poór and Elie Mounzer.

Plan:

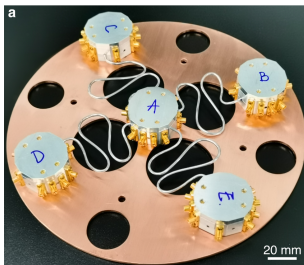
- ▶ Distributed QEC
- ▶ Fault tolerance by construction
- ▶ FTBC for DQEC



VS



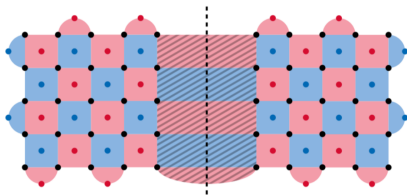
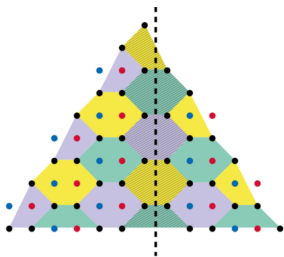
5 module example (Niu et al 2023)



Example numbers:

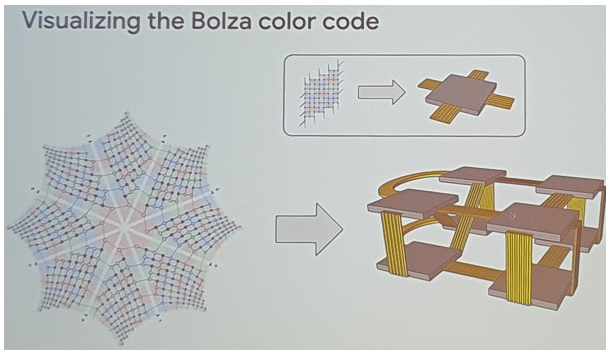
- ▶ 95% Bell fidelity with 60 cm coax (Qiu et al 2023)
- ▶ 96% Bell fidelity with 1m coax (IBM blog post)
- ▶ 99% Bell fidelity with 25cm coax (Niu et al 2023)

Memory and lattice surgery $\rightarrow$ fault-tolerant **distributed** measurement of weight- n stabilizers



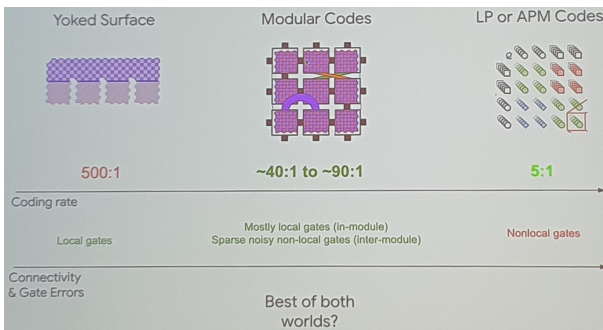
Of recent interest: hyperbolic surface codes and color codes

“Over 10x Denser Fault Tolerance using Boundary-Connected Modules” (Oscar Higgot at QEC26)

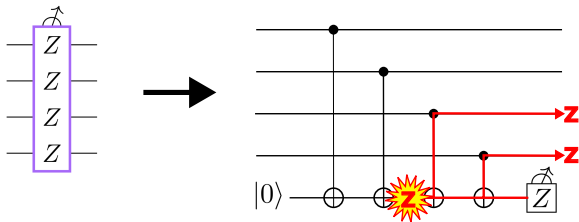


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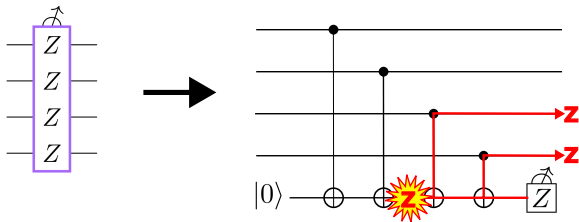
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Require distributed stabilizer measurements between modules



Need implementable circuits using native gate set that don't spread faults, but checking by iterating over faults is costly



Alternative: ZX-calculus ($\subset$ tensor network formalism)

Fault equivalent rewrites $\subset$ ZX-rewrites

→ **fault tolerance by construction**

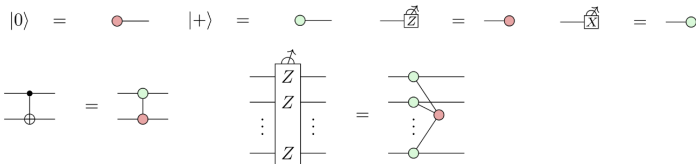
Rodatz et al

arXiv:2506.17181

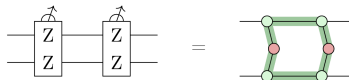


ZX-ingredients

Operations:



Pauli webs for detectors:



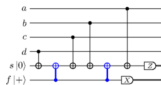
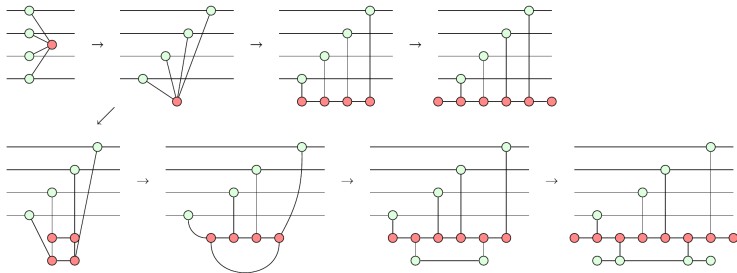
ZX-rewrites: fusion, deforming lines, ...

preserve circuit semantics but not necessarily fault tolerance

A subset of ZX-rewrites preserve behavior under noise. Ex:

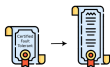


Example rewrite:



Fault Tolerance By Construction for Distributed QEC

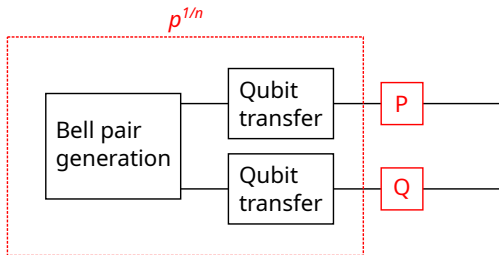
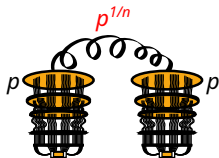
- ▶ Extending the framework



- ▶ Applying it to the noisy Bell pair setting



Simple model:



$p \in [10^{-4}, 10^{-3}]$, $n \in [2, 3]$ gives $p_{\text{Bell}} \in [1\%, 10\%]$.

Circuit representation: bent edges with *weights*

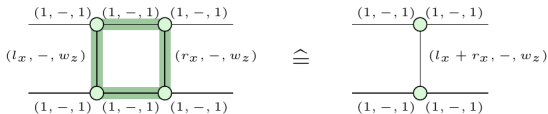


$$(w_x, w_y, w_z) \rightarrow p_x = p^{w_x}, p_y = p^{w_y}, p_z = p^{w_z}$$

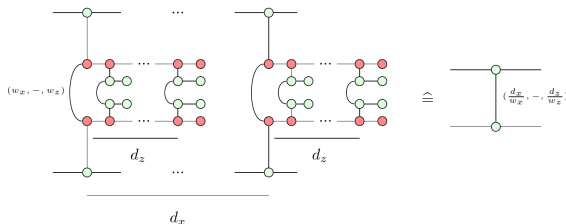
$$(w_x, w_y, w_z)$$

$$p_x = p^{w_x}, p_y = p^{w_y}, p_z = p^{w_z}$$

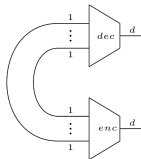
Edge improvement, example 1



Edge improvement, example 2



Like a “spread out” Shor code distillation.
Can do same with other codes.



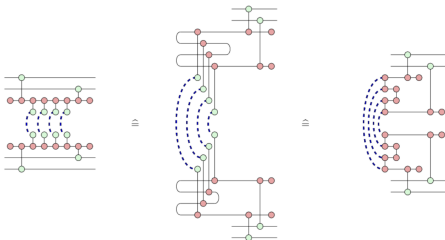
Can plug into weight- n measurement rewrites

Weight 4 and 6 stabilizer measurement examples

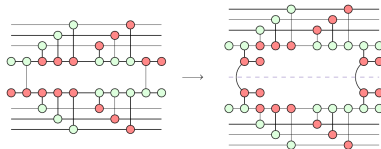


Main gain: **flexibility**

- Space time tradeoffs



- Context aware circuits: some fault propagation is benign

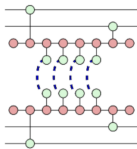


Numerics, example 1. Surface code with:

Circuit level noise

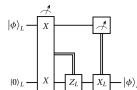
Very noisy Bell pairs: $p_{\text{Bell}} = p^{1/3}$, so $p = 10^{-3} \rightarrow p_{\text{Bell}} = 10^{-1}$

Limited space for auxilliary qubits and limited Bell pair rate
→ maximally sequential setting...

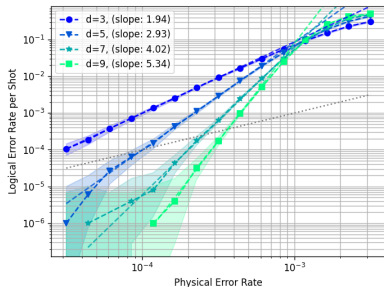


...and idle noise as bad as 2Q while waiting

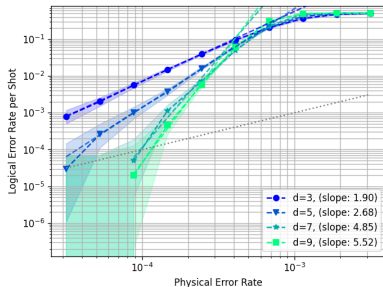




Memory

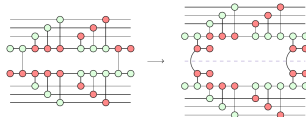


Lattice surgery teleportation

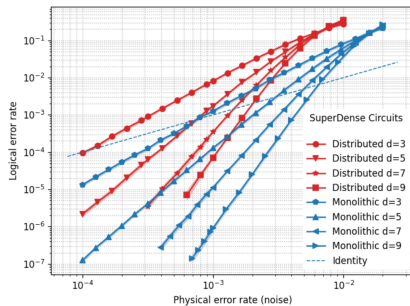
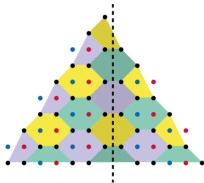


Recover slopes, threshold much lower

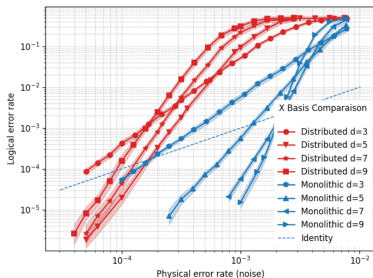
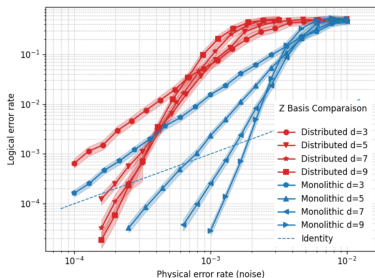
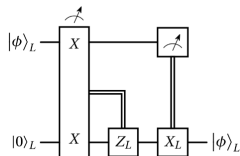
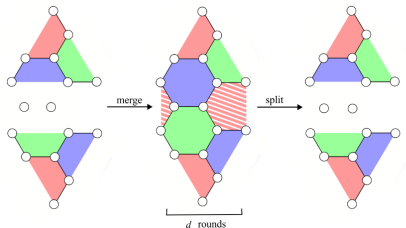
Numerics, example 2: color code, superdense + inner repetitions done in parallel



Memory

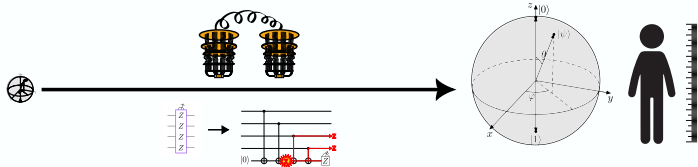


Lattice surgery teleportation



Decoder: Tesseract (short beam). Long beam better, but slow

Summary



- ▶ Large overhead motivates distributed QEC
- ▶ Need implementable, fault-tolerant circuits
- ▶ Can do stabilizer measurements with very noisy Bell pairs

Future:

- ▶ Methods for context awareness
- ▶ Noise models closer to hardware
- ▶ Decoding

Thank you!

